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# Bogdan, Geometry and Quanta

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**Abstrakt.** Omawiamy wybrane osiągnięcia zmarłego niedawno Bogdana Mielnika na polu fizyki teoretycznej, ze szczególnym uwzględnieniem jego prób wyjścia poza standardową mechanikę kwantową. Przypominamy również niektóre z jego poglądów na problemy współczesnego społeczeństwa i organizację nauki.

**Słowa kluczowe:** teoria kwantowa, podstawy fizyki, Bogdan Mielnik

**Abstract.** We review selected achievements of the late Bogdan Mielnik in the field of theoretical physics, with an emphasis on his attempts to go beyond standard quantum mechanics. Some of his original views on the problems of contemporary society and organization of science are also recalled.

**Keywords:** quantum theory, foundations of physics, Bogdan Mielnik

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Bogdan Mielnik (1936-2019) graduated in theoretical physics in 1958 at the University of Warsaw, which at the time was one of the select few places where the renaissance of general relativity as a central field of physics was being prepared. Invited by his supervisor Jerzy Plebański, Bogdan went to Mexico and submitted his PhD thesis [1] on October 22, 1964. He was the very first Ph.D. graduate of the Physics Department at the Center for Research and Advanced Studies of the National Polytechnic Institute Cinvestav founded in Mexico City in 1961. He returned to Poland in 1965. In 1975 he made a memorable visit to Stockholm, one of the quiet places where the theory of open quantum systems saw the light of day. He went again to Mexico in 1981 and afterwards shared his time between the University of Warsaw and Cinvestav. Eventually he described some of his adventures in the Plebański Festschrift [2].

A central theme of Bogdan's work was to understand why quantum mechanics takes the shape it does, and whether it needs to be changed (perhaps when taking gravity into account). When he started out quantum logic gave its answers to the first question, and made it seem as if no changes in the formalism were possible. Bogdan—inspired by Günther Ludwig and by Rudolf Haag, if we read his reference lists correctly—took a more geometrical view of the axioms. Instead of a lattice of propositions his starting point was the more flexible lattice of faces of convex bodies. In the late sixties he published two notable papers [3, 4] in *Commun. Math. Phys.*, in which he explained how the geometry of the set of states of a physical system is determined by the transition probabilities associated to idealized experiments. If we impose suitable 'crystalline symmetries' on the convex body (the *statistical figure* in Bogdan's terminology) the lattice of faces can be identified with the lattice of subspaces in a Hilbert space, and hence quantum logic is recovered.

Here we must admit that when (many years later) we wrote a book on the geometry of quantum theory we borrowed its title [5] from one of these papers. Fascinated by the originality of his thoughts we also opened our book with a quotation from another paper of his [6]: *What picture does one see, looking at a physical theory from a distance, so that the details disappear? Since quantum mechanics is a statistical theory, the most universal picture which remains after the details are forgotten is that of a convex set.*



Fig. 1. The entire scientific career of Bogdan Mielnik was strongly linked to the University of Warsaw and Cinvestav in Mexico City

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Bogdan's pioneering ideas were appreciated later on, when the development of the field of quantum information science made the space of quantum states into a stage on which quantum algorithms can be played.

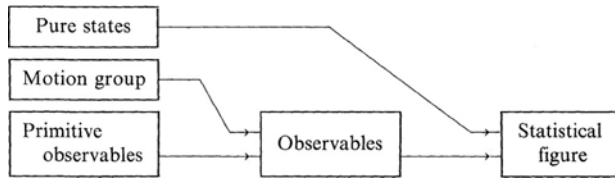


Fig. 2. Sketch of the intrinsic structure of quantum mechanics presented in the paper of Bogdan on generalized quantum theories [10]

In particular Bogdan took an original view of the concept of the state of a system. For him the physical system is always open, and subject to the influences of the universe in which it is placed. It is this *mobility* that defines the system. This means that the one-parameter group transformations generated by a definite Hamiltonian loses its central status. Instead we have to ask for the *motion group*, the set of all transformations that the universe may cause the system to undergo. This leads to the *dynamical manipulation problem* that Bogdan initiated in Stockholm, this time inspired by the works of Willis Lamb and Elihu Lubkin. This problem has a rich technical content in the framework of orthodox quantum mechanics, and it was to occupy much of his attention in later years (although the idea to challenge orthodoxy was still with him, as one can see in his grand review of factorization and supersymmetric quantum mechanics [7]).

With a persistent enthusiasm Bogdan analyzed the interrelations between quantum theory and general relativity. He stressed the flexibility introduced by the latter theory when it went from a fixed spacetime to a dynamically determined one, and he felt—not surprisingly given his background—that joining it to the rigid structure of quantum mechanics may violate the *innate aesthetics of general relativity* [8].

In the end no satisfactory alternative presented itself. And over the past fifty years some very important developments have taken place: Quantum theory has been tested in the laboratory in a wide range of qualitatively new circumstances, and it has never been found wanting. But one of Bogdan's papers [6] starts with a quotation from Stanisław Lem, worth repeating here: *It is well known that dragons do not exist. But each one does it in a different way.* Bogdan presumably quoted from memory, because his version is snappier than the original. These words can be considered as a prophetic prediction of the enormous scientific interest in generalized probabilistic theories developed in the current century, and intended to go beyond standard quantum

mechanics. The contributions in the book edited by Chiribella and Spekkens [9] can be recommended for an update. We mean no offence if we add that for charm, originality, and seriousness, Bogdan's papers still stand supreme.

And concerning quantum gravity, there is as yet no need to make changes to the conclusion of his 1974 paper [10]: *The incompleteness of the present day science at this point is, perhaps, one more reason why the scheme of quantum mechanics should not be prematurely closed.*

In the late spring of 2000 Ingemar was visiting Poland during a time when Bogdan was temporarily working in Warsaw. We met him in the small bar at the old building of the Faculty of Physics at Hoża street, and our discussion turned out to be inspiring and unforgettable. We recall the details after 20 years. Sipping barszcz (beet root soup) we learned more about the key ideas of Bogdan on what he referred to as the game of quantum mechanical pick-a-stick (or 'jackstraws'—none of us knew the English term). Others know this game as the problem of quantum control [11]. We also learned about his ideas concerning nonlinear quantum mechanics [12] and, while savouring pierogi, we could appreciate his novel approach to the Floquet theory [13, 14] of time dependent quantum systems. Bogdan's story was told in such a convincing and emotional way that we felt that we should put aside all current projects and start working on this very topic.

A decade later all three of us met again in Białowieża at the 2011 workshop *Geometric Methods in Physics*, organized by Anatol Odziejewicz<sup>1</sup> and his colleagues from Białystok and Mexico to celebrate the 75-th birthday of Bogdan. It was a remarkable experience to attend several talks explaining the contribution of Bogdan to our understanding of quanta, and to hear Bogdan himself describe the state of the art of his programme to explore the role of convex geometry at the root of physics [15]. Furthermore, we had a unique chance to meet a crowd of Bogdan's friends and former students from Mexico, and to see how much they admire him and his achievements.

Getting to know the Mexican friends of Bogdan in Białowieża Karol was also admitted to the clan and invited to attend a *Quantum Fest* event in Mexico City. During this conference Bogdan presented his view on Non-Inertial Quantization [16]. It was a remarkable experience to watch Bogdan in action in his Spanish-speaking environment. Each time he appeared in the Institute or

1. Professor Anatol Odziejewicz passed away on April 18th 2022 (editor's note).

entered a conference room or even a dining room he immediately attracted attention, as numerous people wanted to hear what he had to say. It was clear to us that Bogdan Mielnik has played a key role in the development of theoretical physics in Mexico<sup>2</sup>. A brief review of his activity there was presented by Fernandez [17].



Fig. 3. Bogdan Mielnik during the conference *Quantum Fest*, Tecnológico de Monterrey, Campus Estado de México, October 2015 (fot. Marco Enriquez)

Visiting Mexico City for the very first time is a kind of an adventure even for an accomplished globetrotter. One can be amazed even looking at the traffic jams in broad streets with several lanes, which are not resolved by highways constructed in parallel, the upper lanes just above the lower lanes. For any visitor driving in this city looks as a real challenge and orientation without any navigation devices seems impossible.

Bogdan knew the city very well and he explained to us his way of using cabs, he mastered far before the age of smartphones. *You do not tell the driver your final destination. You just look around and keep saying: La derecha, por favor. Gire a la izquierda aquí, until you reach your destination. This is simple. And is really safe.* It looks like the life in Mexico City is not very easy, unless one gets used to it. As Bogdan did, as he really enjoyed his stay in Mexico and his work in this culture.

We had a privilege to discuss with Bogdan numerous issues far beyond current problems of quantum theory.

2. It is worth mentioning that Bogdan has an extensive wikipedia webpage in the Spanish version, but up till now, he has none in Polish nor in English.

In particular we were interested to learn his original opinions on contemporary society, organization of science [18] and recent economic problems [19]. To give reader a glimpse of his thoughts in this direction we present in the Appendix some quotations from the article on bureaucracy in science [18].

## Appendix

### Bogdan's thoughts on the Bureaucratic World

Selected quotations from the paper of Bogdan [18] are presented below.

*Esteemed Colleagues: The remarks below concern a certain lack of equilibrium in the present day legislation, affecting the life and science, with rather adverse consequences for our work.*

*The damage to science consists not only in our loss of time, but much more in the fact that the scientist of today is forced to subordinate himself to some counter-intellectual patterns of reports and planning, forcing him indeed to accept the professional dishonesty. The most absurd demand he faces is to present the program (and the time-table) of his future discoveries. Such plans can bring the best results if they fail ... The excursion of Christopher Columbus [did not] accomplish his original plan to discover the shortest way to India. The only thing discovered by CC was an obstacle, on which we live today!*

*New forms of business appear: the enterprises which help the scientists to formulate their grant requests in terms convincing for the bureaucrats. (The corruptive consequences are not difficult to guess!)*

### Four Laws of Bureaucracy

*I. All attempts of the state administrations to improve the scientific work by bureaucratic projects, reports, etc. will be reduced to zero by the social organism – though not gratis: the price is an enormous increase of socially useless work.*

*II. What is the source of the incredible facility of public administrations in multiplying endlessly the prescriptions, formalities and obligatory documents? The reason is that the bureaucrats do not perform the bureaucratic work: they leave it to their victims.*

*III. In the bureaucratic environment the problems of little importance are always infinitely more urgent than the truly important ones. This is why thou will never do anything important.*

*IV. The knowledge of the four bureaucracy laws won't help you in anything.*

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